Spatio-Temporal Surrogate Construction and Calibration of E3SM Land Model

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Motivation: Uncertainties in Carbon Flux

Overview: Surrogate-based Calibration of E3SM Land Model

- Land-surface model parametric uncertainty remains large
- High model expense \rightarrow Need for model surrogates for sample-intensive studies, such as ...
	- Global sensitivity analysis (forward UQ)
	- Model calibration (inverse UQ)
- Major challenges
	- Expensive model evaluation, small ensembles
	- High dimensional (spatio-temporal) outputs
- Reduced-dimensional, inexpensive surrogate construction via Karhunen-Loève expansions and Neural Networks (**KLNN surrogate**)
- Surrogate enables global sensitivity analysis and **Bayesian model calibration**

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E3SM Land Model (ELM)

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Model Ensemble (275 samples)

Perturbed Parameters

2000 January 60°N $30°N$ 0° 30°S 60°S $90°5$

150°W $120^{\circ}E$ 150°E 120°W 90° W 60°W 30°W 0° $30^{\circ}E$ 60°E 90°E $180°$

 $\overset{\circ}{\mathsf{GPP}}$ Mean 0.0 1.5 3.0 4.5 $6,0$ 9.0 10.5 12.0 13.5

Forward UQ

a.k.a. surrogate construction, global sensitivity analysis, uncertainty propagation

Dimensionality Reduction via Karhunen-Loève Expansion

 $f(\lambda; z) \approx f(z) + \sum$ $m=1$ \overline{M} $\xi_m(\lambda)\sqrt{\mu_m}\phi_m(z)$

Uncertain parameters "Certain" conditions

- Spatio-temporal model output $f(\lambda; z)$, where $z = (x, y, t)$
- Output field has large dimensionally $N = N_x \times N_y \times N_t$
- Eigenpairs ($\mu_m, \phi_m(z)$) are found via eigen-solve
- Analysis reduces to $M \ll N$ eigenfeatures $\xi_1, ..., \xi_m$
- Under the hood: this is essentially an SVD

KL is essentially a Singular Value Decomposition

KL
$$
f(\lambda^k; z_i) - \overline{f}(z_i) \approx \sum_{m=1}^M \xi_m(\lambda^k) \sqrt{\mu_m} \phi_m(z_i)
$$

\n
$$
F_{ki} = \sum_{m=1}^M U_{km} \Sigma_{mm} V_{im}
$$
\nSVD $F = U \Sigma V^T$

Karhunen-Loève expansion

- -– is centralized (first subtract the mean)
- -– often comes with the continuous form
- -– has random variable interpretation for the latent features (aka left singular vectors) ξ_m

KL truncation relies on variance retention

$$
f(\lambda; z) \approx \overline{f}(z) + \sum_{m=1}^{M} \xi_m(\lambda) \sqrt{\mu_m} \phi_m(z)
$$

$$
Var[f(z)] = \sum_{m=1}^{M} \mu_m \phi^2_m(z)
$$

$$
M = \operatorname{argmin}_{M'} \frac{\sum_{m=1}^{M'}}{\sum_{m=1}^{\infty} \mu_m} > 0.99
$$

$$
\sum_{m=1}^{M} \mu_m
$$

$$
Var[f] = \sum_{m=1}^{M} \mu_m
$$

KL+NN = reduced dimensional spatio-temporal surrogate

The goal is to construct a surrogate with respect to uncertain parameters λ , such that $f(\lambda; z_i) \approx f_s(\lambda; z_i)$ for all conditions z_i .

Instead of building surrogate for each individual z_i for $i = 1, ..., N$, we construct neural network (NN) surrogate for $\xi_1, ..., \xi_M$ where $M \ll N$.

Selected set of 96 FLUXNET sites

… selected to represent a range of Plant Functional Types (PFTs)

Case studies

Dimensionality reduction via KL

Per-site dimensionality reduction **Per-PFT** dimensionality reduction

Two randomly

KL+NN: two levels of approximations

KL+NN surrogate performance

Instead of 96x180=**17280** surrogates, we build a single NN surrogate in the reduced, **8**-dimensional latent space

Sensitivity at 96 FLUXNET sites: RuBisCO leaf fraction (fLNR) is the most impactful param.

Case studies

Dimensionality reduction from 4000 cells x 4 seasons = **16000** to **11**-dimensional latent space

ELM Model Samples KLNN Surrogate Samples

fLNR sensitivity across the globe

mbbopt sensitivity across the globe

Inverse UQ

a.k.a. calibration or parameter estimation

Reference Data

FLUXCOM: A gridded GPP benchmark upscaled from FLUXNET network using meteorology, remote sensing

https://www.fluxcom.org/

 $p(\lambda | g) \propto p(g|\lambda) p(\lambda)$ Bayes' formula

Bayesian Likelihood is constructed in the reduced space

Bayes' formula $p(\lambda|g) \propto p(g|\lambda)p(\lambda)$

 $f(\lambda; z) \approx f(z) + \sum$ $m=1$ \overline{M} $\xi_m^{NN}(\lambda)\sqrt{\mu_m}\phi_m(z)$ KLNN surrogate:

Project observed data to the KL eigenspace:

$$
g(z) \approx \overline{f}(z) + \sum_{m=1}^{M} \eta_m \sqrt{\mu_m} \phi_m(z)
$$

Pointwise likelihood (naïve) : '

$$
L_g(\lambda) \equiv p(g|\lambda) \propto \exp\left(-\sum_{i=1}^N \frac{(g(z_i) - f(\lambda; z_i))^2}{2\sigma_i^2}\right)
$$

2 **Reduced likelihood :** '

$$
L_g(\lambda) \equiv p(g|\lambda) \propto \exp\left(-\sum_{m=1}^M \frac{(\eta_m - \xi_m^{NN}(\lambda))^2}{2\sigma^2}\right)
$$

Eigenfeatures ξ_m 's are uncorrelated, zero-mean, unit variance, hence iid gaussian likelihood is a much better assumption in the reduced space.

Surrogate-enabled calibration workflow incorporates both forward and inverse UQ tasks

Latent space distance is well-correlated with the physical distance between model and data **Model-Data RMSE**

US-Ha1

US-GLE

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Bayesian MCMC calibration enabled by KLNN surrogate

J.S. DEPARTMENT OF 27

US-MOz 17.5 -- Fluxnet data **Model Prior Model Posterior** 15.0 12.5 10.0 GPP 7.5 5.0 2.5 0.0 US-Ha1 17.5 - Fluxnet data **Model Prior Model Posterior** 15.0 12.5 $\left[\begin{matrix}10.0\\ 0\\ 7.5\end{matrix}\right]$ 7.5 5.0 2.5 0.0 2002-2005-2000-2003-2006 2009 2010 2012-

2002

2008

2021

2013

2014

2007

2004

Time evolution of GPP at select FLUXNET sites

Calibration brings model prediction closer to reference data

Site-specific parameters

Sites

Nominal parameter (prior) Max a posteriori (MAP) Reference data

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Summary

- Karhunen-Loève (KL) decomposition reduces the spatio-temporal output dimensionality, taking advantage of correlations over space and time.
- Neural network (NN) surrogate in the reduced eigenspace leads to a spatio-temporal KLNN surrogate that is a small fraction of ELM cost.
- KLNN surrogate enables sampling based global sensitivity analysis and Bayesian calibration performed in the eigenspace.

Ongoing work:

- *Potential PFT-dependent reparameterization to improve model's ability to match reference data.*
- *Calibration with embedded model discrepancy to avoid overfitting.*

Additional Material

Motivation: Model Uncertainty dominates for Land Model

Fig. 4. Ocean and land carbon cycle

uncertainty. The percentage of total variance attributed to internal variability, model uncertainty, and scenario uncertainty in projections of cumulative global carbon uptake from 2006 to 2100 differs widely between (A) ocean and (B) land. The ocean carbon cycle is dominated by scenario uncertainty by the middle of the century, but uncertainty in the land carbon cycle is mostly from model structure. Data are from 12 ESMs using four different scenarios (94).

Bonan and Doney,

Climate, ecosystems, and planetary futures: The challenge to predict life in Earth system models. Science, 2018

Polynomial Chaos intro

- Our traditional tool for uncertainty representation and propagation
- Random variables represented as polynomial expansion of standard random variables, such as gaussian or uniform \boldsymbol{K} $c_k \psi_k(\eta)$

 $k=1$

• Convenient for uncertainty propagation

$$
f(\xi) = \sum_{k=0}^{K} f_k \psi_k(\eta)
$$

- Moment estimation
- Global Sensitivity Analysis (a.k.a. Sobol indices or variance-based decomposition)

KL+PC = reduced dimensional spatio-temporal surrogate

The goal is to construct a surrogate with respect to uncertain parameters λ , such that $f(\lambda; z_i) \approx f_s(\lambda; z_i)$ for all conditions z_i .

Instead of building surrogate for each individual z_i for $i = 1, ..., N$, we construct polynomial chaos (PC) surrogate for $\xi_1, ..., \xi_M$ where $M \ll N$.

$$
f(\lambda; z) \approx \overline{f}(z) + \sum_{m=1}^{M} \xi_m(\lambda) \sqrt{\mu_m} \phi_m(z)
$$

$$
\xi_m^{PC}(\lambda)
$$

PC vs NN comparison

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PC vs NN comparison

96 temporal surrogates with each 180 outputs

Single spatio-temporal surrogate with 96x180 outputs

Likelihood in the reduced space is still Gaussian, but MVN

Local (site-specific) fLNR posterior PDFs

Grouped by PFTs

Two calibration regimes

Fixed global fLNR parameter **Local fLNR** parameter

One global surrogate One surrogate per grid cell

Localized calibration works slightly better

Correlate PFT fractions globally with best fLNR values

PFT Fractions for all PFTs

Correlate PFT fractions globally with best fLNR values

